A $\quad a = 1$

B $\quad b = -1$

$$z' = \frac{z-1}{z+1} \qquad z' = f(z)$$

1) <u>Points invariants de f :</u>

On cherche z tels que $z' = z$.

$$\frac{z-1}{z+1} = z$$

$$z(z+1) = z - 1$$

$$z^2 + z = z - 1$$

$$z^2 = -1$$

donc $z = i$ ou $z = -i$

donc 2 points invariants le point d'affixe i et le point d'affixe $-i$.

2) a) <u>Montrer que $(z'-1)(z+1) = -2$</u>

$$(z'-1)(z+1) = \left(\frac{z-1}{z+1} - 1\right)(z+1) = \frac{\cancel{z}-1-\cancel{z}-1}{\cancel{z+1}} \times (\cancel{z+1}) = -2$$

b) $|z'-1|\,|z+1| = |-2|$

$|z'-1|\,|z+1| = 2$

$|z'-a|\,|z-b| = 2$

$\boxed{AM' \times BM = 2}$

$\mathrm{Arg}(z'-1) + \mathrm{Arg}(z+1) = \mathrm{Arg}(-2)$

$\mathrm{Arg}(z'-a) + \mathrm{Arg}(z-b) = \pi$

$\boxed{(\vec{u}, \overrightarrow{AM'}) + (\vec{u}, \overrightarrow{BM}) = \pi}$

les angles $(\vec{u}, \overrightarrow{AM'})$ et $(\vec{u}, \overrightarrow{BM})$ sont supplémentaires

3) Si $M \in \mathcal{C}(B, 2)$ alors $BM = 2$

donc $AM' = \frac{2}{BM} = 1$

donc $M' \in \mathcal{C}(A, 1)$

b) $z_C = 2$

Si $M \in \mathcal{C}(C, 2)$ alors $CM = 2$

c.a.d $|z_M - z_C| = 2$

c.a.d $|z - 2| = 2$

On a : $z' - 3 - 2i = (1+i)(z-2)$

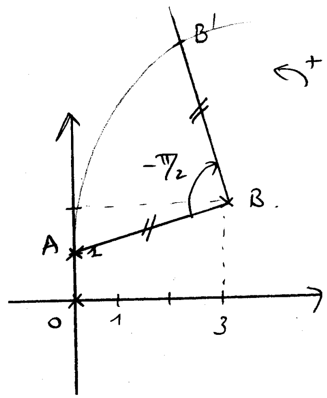
donc $|z' - (3+2i)| = |1+i|\,|z-2|$

donc $|z' - (3+2i)| = \sqrt{2} \times 2$

$|z' - z_B| = 2\sqrt{2}$

$BM' = 2\sqrt{2}$

donc $M' \in \mathcal{C}(B, 2\sqrt{2})$



$BB' = AB$

$(\overrightarrow{BA}, \overrightarrow{BB'}) = -\frac{\pi}{2}$