DM Intégrales.

Ex 1

1) $I = \int_0^1 \frac{3x}{4x^2+1}\,dx$

$I = [F(x)]_0^1$

$= \left[\frac{3}{8}\ln(4x^2+1)\right]_0^1$

$= \frac{3}{8}\ln(5) - \frac{3}{8}\ln(1)$

$\boxed{I = \frac{3}{8}\ln 5}$

$f(x) = \frac{3x}{4x^2+1}$

formule $\frac{u'}{u}$ avec $u(x) = 4x^2+1$

$u'(x) = 8x$

$f(x) = \frac{3}{8} \times \frac{8x}{4x^2+1}$

$\boxed{F(x) = \frac{3}{8}\ln(4x^2+1)}$

car $4x^2+1>0$ sur $[0,1]$

2) $J = \int_1^2 \frac{x}{(3x^2+1)^2}\,dx$

$J = [F(x)]_1^2$

$J = \left[-\frac{1}{18x^2+6}\right]_1^2$

$= -\frac{1}{78} + \frac{1}{24}$

$\boxed{J = \frac{3}{104}}$

$f(x) = \frac{x}{(3x^2+1)^2}$

formule $\frac{u'}{u^2}$ avec $u(x) = 3x^2+1$

$u'(x) = 6x$

$f(x) = \frac{1}{6} \times \frac{6x}{(3x^2+1)^2}$

$F(x) = \frac{1}{6} \times \frac{-1}{3x^2+1}$

$\boxed{F(x) = \frac{-1}{18x^2+6}}$

3) $K = \int_0^1 x e^{-x^2}\,dx$

$K = [F(x)]_0^1$

$K = \left[-\frac{1}{2}e^{-x^2}\right]_0^1$

$K = -\frac{1}{2}e^{-1} + \frac{1}{2}e^0$

$\boxed{K = -\frac{1}{2}e^{-1} + \frac{1}{2}}$

$\boxed{K = -\frac{1}{2e} + \frac{1}{2}}$

$f(x) = x e^{-x^2}$

formule $u'e^u$ avec $u(x) = -x^2$

$u'(x) = -2x$

$f(x) = \frac{1}{-2} \times (-2x)\,e^{-x^2}$

$\boxed{F(x) = -\frac{1}{2}e^{-x^2}}$

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Ex 2 $f(x) = \frac{3}{\sqrt{20-4x}}$

Sur $[1,4]$ $f(x) \geq 0$ donc Aire $= \int_1^4 f(x)\,dx$

$\frac{u'}{\sqrt{u}}$	$2\sqrt{u}$
$\frac{-4}{\sqrt{20-4x}}$	$2\sqrt{20-4x}$

$f(x) = \frac{3}{-4} \times \frac{-4}{\sqrt{20-4x}}$

$F(x) = -\frac{3}{4} \times 2\sqrt{20-4x}$

$\boxed{F(x) = -\frac{3}{2}\sqrt{20-4x}}$

Aire $= \left[-\frac{3}{2}\sqrt{20-4x}\right]_1^4$

$= -\frac{3}{2}\sqrt{4} + \frac{3}{2}\sqrt{16}$

$= -\frac{3}{2}\times 2 + \frac{3}{2}\times 4 = -3+6 = 3$

$\boxed{\text{Aire} = 3\text{ U.A}}$

$1\text{ U.A} = 1{,}5 \times 0{,}5\text{ cm}^2$

$= 0{,}75\text{ cm}^2$

$= \frac{3}{4}\text{ cm}^2$

Aire $= 3 \times 0{,}75\text{ cm}^2$

Aire $= 3 \times \frac{3}{4}\text{ cm}^2$

$= \frac{9}{4}\text{ cm}^2$

$\boxed{\text{Aire} = 2{,}25\text{ cm}^2}$