Ex Soit $I = \int_2^3 \frac{x}{x^2-1}\,dx \qquad J = \int_2^3 \frac{1}{x^2-1}\,dx$

1) Calculer I.

$$I = \int_2^3 \frac{1}{2}\,\frac{2x}{x^2-1}\,dx = \frac{1}{2}\left[\ln(x^2-1)\right]_2^3$$

car sur $[2,3]$ $x^2-1>0$

x		-1		1	
x^2-1	$+$	0	$-$	0	$+$

$I = \frac{1}{2}(\ln 8 - \ln 3)$

$$\boxed{I = \frac{1}{2}\ln\left(\frac{8}{3}\right)}$$

2) Calculer $I+J$ puis en déduire J.

$$I+J = \int_2^3 \frac{x+1}{x^2-1}\,dx = \int_2^3 \frac{x+1}{(x+1)(x-1)}\,dx$$

$$= \int_2^3 \frac{1}{x-1}\,dx \quad \text{car } x-1>0 \text{ sur } [2,3]$$

$$= \left[\ln(x-1)\right]_2^3 = \ln 2 - \ln 1 = \boxed{\ln 2}$$

donc $J = \ln 2 - I$

$$J = \ln 2 - \frac{1}{2}\ln\left(\frac{8}{3}\right) = \ln 2 - \ln\left(\sqrt{\frac{8}{3}}\right)$$

$$= \ln 2 - \ln\left(\frac{\sqrt{8}}{\sqrt{3}}\right) = \ln\left(\frac{2}{\frac{\sqrt{8}}{\sqrt{3}}}\right)$$

$$= \ln\left(\frac{2\times\sqrt{3}}{\sqrt{8}}\right) = \ln\left(\frac{2\sqrt{3}}{2\sqrt{2}}\right)$$

$$= \ln\left(\sqrt{\frac{3}{2}}\right)$$

$$= \boxed{\frac{1}{2}\ln\left(\frac{3}{2}\right)}$$