Calculs d'intégrales - Correction

Calculer les intégrales suivantes :

1. $I = \displaystyle\int_0^1 \mathrm{e}^{5x+2}\,\mathrm{d}x = \int_0^1 \frac{1}{5} \times 5\mathrm{e}^{5x+2}\,\mathrm{d}x = \frac{1}{5}\int_0^1 5\mathrm{e}^{5x+2}\,\mathrm{d}x = \frac{1}{5}\int_0^1 u'(x)\mathrm{e}^{u(x)}\,\mathrm{d}x$

$I = \dfrac{1}{5}\left[\mathrm{e}^{u(x)}\right]_0^1 = \dfrac{1}{5}\left[\mathrm{e}^{5x+2}\right]_0^1 = \boxed{\dfrac{1}{5}\left(e^7 - \mathrm{e}^2\right)}$

2. $J = \displaystyle\int_1^3 \frac{1}{x^2}\left(\frac{3}{x}-2\right)^2\,\mathrm{d}x = \int_1^3 \frac{1}{-3} \times \frac{-3}{x^2}\left(\frac{3}{x}-2\right)^2\,\mathrm{d}x = -\frac{1}{3}\int_1^3 \frac{-3}{x^2}\left(\frac{3}{x}-2\right)^2\,\mathrm{d}x = -\frac{1}{3}\int_1^3 u'(x)\left(u(x)\right)^2\,\mathrm{d}x$

$J = -\dfrac{1}{3}\left[\dfrac{1}{3}(u(x))^3\right]_1^3 = -\dfrac{1}{3} \times \dfrac{1}{3}\left[(u(x))^3\right]_1^3 = -\dfrac{1}{9}\left[\left(\dfrac{3}{x}-2\right)^3\right]_1^3 = -\dfrac{1}{9}\left((-1)^3 - 1^3\right) = \boxed{\dfrac{2}{9}}$

3. $K = \displaystyle\int_0^1 \frac{x}{3x^2+1}\,\mathrm{d}x = \int_0^1 \frac{1}{6} \times \frac{6x}{3x^2+1}\,\mathrm{d}x = \frac{1}{6}\int_0^1 \frac{u'(x)}{u(x)}\,\mathrm{d}x$

$K = \dfrac{1}{6}\left[\ln\,(u(x))\right]_0^1$ car $u(x) = 3x^2+1 > 0$ sur $[0\,;\,1]$

$K = \dfrac{1}{6}\left[\ln\left(3x^2+1\right)\right]_0^1 = \dfrac{1}{6}\left(\ln 4 - \ln 1\right)) = \boxed{\dfrac{1}{6}\ln 4}$

4. $L = \displaystyle\int_0^\pi \frac{\cos x}{(3\sin x + 2)^2}\,\mathrm{d}x = \int_0^\pi \frac{1}{3} \times \frac{3\cos x}{(3\sin x+2)^2}\,\mathrm{d}x = \frac{1}{3}\int_0^\pi \frac{u'(x)}{(u(x))^2}\,\mathrm{d}x$

$L = \dfrac{1}{3}\left[\dfrac{-1}{u(x)}\right]_0^\pi = \dfrac{1}{3}\left[\dfrac{-1}{3\sin x+2}\right]_0^\pi = \dfrac{1}{3}\left(\dfrac{-1}{3\sin\pi+2} - \dfrac{-1}{3\sin 0+2}\right) = \dfrac{1}{3}\left(\dfrac{-1}{2} - \dfrac{-1}{2}\right) = \boxed{0}$

5. $L = \displaystyle\int_0^1 \frac{\mathrm{e}^{-2x}}{\sqrt{\mathrm{e}^{-2x}+1}}\,\mathrm{d}x = \int_0^1 \frac{1}{-2} \times \frac{-2\mathrm{e}^{-2x}}{\sqrt{\mathrm{e}^{-2x}+1}}\,\mathrm{d}x = \frac{-1}{2}\int_0^1 \frac{u'(x)}{\sqrt{u(x)}}\,\mathrm{d}x$

$L = \dfrac{-1}{2}\left[2\sqrt{u(x)}\right]_0^1 = \dfrac{-1}{2}\left[2\sqrt{\mathrm{e}^{-2x}+1}\right]_0^1 = \dfrac{-1}{2} \times 2\left[\sqrt{\mathrm{e}^{-2x}+1}\right]_0^1 = -\left(\sqrt{\mathrm{e}^{-2}+1} - \sqrt{\mathrm{e}^0+1}\right)$

$L = -\left(\sqrt{\mathrm{e}^{-2}+1} - \sqrt{2}\right) = \boxed{\sqrt{2} - \sqrt{\mathrm{e}^{-2}+1}}$