Ex 2 $a_1 = \frac{1}{2}$ $\quad a_{n+1} = \frac{n+1}{2n} a_n$ pour $n \geq 1$

1) $a_2 = \frac{2}{2\times 1} a_1 = a_1 = \boxed{\frac{1}{2}}$

$a_3 = \frac{3}{2\times 2} a_2 = \frac{3}{4} \times \frac{1}{2} = \boxed{\frac{3}{8}}$

2) $\forall n \geq 1 \quad b_n = \frac{a_n}{n}$

a) $b_{n+1} = \frac{a_{n+1}}{n+1} = \frac{1}{n+1} \times a_{n+1} = \frac{1}{\cancel{n+1}} \times \frac{\cancel{n+1}}{2n} a_n$

$= \frac{a_n}{2n} = \frac{1}{2} \times \frac{a_n}{n} = \frac{1}{2} \times b_n$

$\boxed{b_{n+1} = \frac{1}{2} b_n}$ donc (b_n) est géométrique de raison $\frac{1}{2}$ de premier terme.

$b_1 = \frac{a_1}{1} = \frac{1}{2}$

b) $\forall n \geq 1 \quad b_n = b_1 \times q^{n-1}$

$b_n = \frac{1}{2} \times \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n$

$\boxed{b_n = \left(\frac{1}{2}\right)^n}$

$b_n = \frac{a_n}{n}$ donc $a_n = n b_n$

$\boxed{a_n = n\left(\frac{1}{2}\right)^n}$

Ex 1 $U_0 = 2 \quad \forall n \geq 0 \quad U_{n+1} = \frac{U_n}{U_n + 1}$

et $V_n = \frac{1}{U_n}$

1) $V_{n+1} - V_n = \frac{1}{U_{n+1}} - \frac{1}{U_n} = \frac{U_n + 1}{U_n} - \frac{1}{U_n} = \frac{U_n}{U_n} = 1$

donc $\boxed{V_{n+1} = V_n + 1}$ pour tout $n \geq 0$.

donc (V_n) est arithm. de raison 1

2) $V_0 = \frac{1}{U_0} = \frac{1}{2}$

donc pour tout $n \geq 0 \quad V_n = V_0 + nr$

$\boxed{V_n = \frac{1}{2} + n}$

$U_n = \frac{1}{V_n}$ donc $U_n = \frac{1}{\frac{1}{2} + n} = \frac{1}{\frac{1+2n}{2}}$

$\boxed{U_n = \frac{2}{1+2n}}$