Sujet A / B.

Q1 $x^2 - 1 - 3x(x-1)$

$= (x-1)(x+1) - 3x(x-1)$

$= (x-1)(x+1-3x)$

$= (x-1)(-2x+1)$

Q2 $A = \dfrac{25^3 \times 5^4}{5^7} = \dfrac{(5^2)^3 \times 5^4}{5^7} = \dfrac{5^6 \times 5^4}{5^7} = \dfrac{5^{10}}{5^7} = \boxed{5^3}$

Q3 $B = \dfrac{5^{n+1} - 5^n}{3^{n+2} - 3^{n-1}} = \dfrac{5^n(5-1)}{3^n(3^2 - 3^{-1})} = \dfrac{5^n}{3^n} \times \dfrac{4}{9 - \frac{1}{3}}$

$B = \left(\dfrac{5}{3}\right)^n \times \dfrac{4}{\frac{26}{3}} = \left(\dfrac{5}{3}\right)^n \times 4 \times \dfrac{3}{26} = \left(\dfrac{5}{3}\right)^n \times \dfrac{2 \times 3}{13}$

$\boxed{B = \left(\dfrac{5}{3}\right)^n \times \dfrac{6}{13}}$

Q4 $C_n = -2 + \dfrac{3}{n} = \dfrac{-2n+3}{n}$

donc $\boxed{\dfrac{1}{C_n} = \dfrac{n}{-2n+3}}$

Q5 $5x^2 = x$

$5x^2 - x = 0$

$x(5x-1) = 0$

$x = 0$ ou $5x - 1 = 0$

$x = \dfrac{1}{5}$

$\boxed{S = \{0 ; \tfrac{1}{5}\}}$

Q6 $\begin{cases} 2x - 3y = 1 \\ -5x + 2y = 2 \end{cases}$

$\Leftrightarrow \begin{cases} x = -\dfrac{8}{11} \\ y = -\dfrac{9}{11} \end{cases}$

$2L_1 : 4x - 6y = 2$

$3L_2 \quad -15x + 6y = 6$

$2L_1 + 3L_2 \quad -11x = 8$

$x = -\dfrac{8}{11}$

Calcul de y :

$2x - 3y = 1$

$-3y = 1 - 2x$

$-3y = 1 + \dfrac{16}{11}$

$-3y = \dfrac{27}{11}$

$\boxed{y = -\dfrac{9}{11}}$

Q7 $f(x) = 3\sqrt{x} + \dfrac{2}{x}$

$y = f'(4)(x-4) + f(4)$

$y = \dfrac{5}{8}(x-4) + \dfrac{13}{2}$

$y = \dfrac{5}{8}x - \dfrac{5}{2} + \dfrac{13}{2}$

$y = \dfrac{5}{8}x + \dfrac{8}{2}$

$\boxed{y = \dfrac{5}{8}x + 4}$

$f(4) = 3\sqrt{4} + \dfrac{2}{4}$

$f(4) = 6 + \dfrac{1}{2} = \dfrac{13}{2}$

$f'(x) = 3 \times \dfrac{1}{2\sqrt{x}} + 2 \times \dfrac{-1}{x^2}$

$f'(4) = \dfrac{3}{2 \times 2} - \dfrac{2}{16}$

$= \dfrac{3}{4} - \dfrac{1}{8}$

$= \dfrac{5}{8}$

Q8 $f(x) = \dfrac{2x^3 + x + 1}{2x - 1}$

$f'(x) = \dfrac{(6x^2+1)(2x-1) - (2x^3+x+1) \times 2}{(2x-1)^2}$

$f'(x) = \dfrac{12x^3 - 6x^2 + \cancel{2x} - 1 - 4x^3 - \cancel{2x} - 2}{(2x-1)^2}$

$f'(x) = \dfrac{8x^3 - 6x^2 - 3}{(2x-1)^2}$

Q9 $U_n = U_1 + (n-1)r$

$U_n = 3 + (n-1) \times 4$

$U_n = 3 + 4n - 4$

$\boxed{U_n = 4n - 1}$

Q10 $U_0 = 2 \quad \forall n \geq 0 \quad U_{n+1} = \dfrac{U_n}{5} = U_n \times \dfrac{1}{5}$

donc (U_n) est géométrique de raison $\dfrac{1}{5}$

1) $U_n = U_0 \times q^n \qquad U_n = 2 \times \left(\dfrac{1}{5}\right)^n$

2) $U_0 + U_1 + \dots + U_n = U_0 \times \dfrac{1 - q^{n+1}}{1 - q} = 2 \times \dfrac{1 - \left(\frac{1}{5}\right)^{n+1}}{1 - \frac{1}{5}}$

$= 2 \times \dfrac{1 - \left(\frac{1}{5}\right)^{n+1}}{\frac{4}{5}} = 2 \times \dfrac{5}{4}\left(1 - \left(\dfrac{1}{5}\right)^{n+1}\right)$

$= \boxed{\dfrac{5}{2}\left(1 - \left(\dfrac{1}{5}\right)^{n+1}\right)}$

Q11 $S_n = \displaystyle\sum_{k=1}^{n} \dfrac{1}{k^2}$

1) $S_2 = \displaystyle\sum_{k=1}^{2} \dfrac{1}{k^2} = \dfrac{1}{1^2} + \dfrac{1}{2^2} = 1 + \dfrac{1}{4} = \boxed{\dfrac{5}{4}}$

2) $\boxed{S_{n+1} = S_n + \dfrac{1}{(n+1)^2}}$