Maths Compl.

## Ex1

1) $\dfrac{9^{n+1}}{3^n} = \dfrac{(3^2)^{n+1}}{3^n} = \dfrac{3^{2n+2}}{3^n} = \boxed{3^{n+2}}$

2) $U_n = \dfrac{7^n}{2^{2n+1}}$

$U_{n+1} = \dfrac{7^{n+1}}{2^{2(n+1)+1}} = \dfrac{7^{n+1}}{2^{2n+3}} = \dfrac{7^n \times 7}{2^{2n+1} \times 2^2}$

$\boxed{U_{n+1} = U_n \times \dfrac{7}{4}}$

3) a) $V_n = \dfrac{7^n}{n!}$ $\quad V_{n+1} = \dfrac{7^{n+1}}{(n+1)!} = \dfrac{7^n \times 7}{n! \times (n+1)}$

$\boxed{V_{n+1} = V_n \times \dfrac{7}{n+1}}$

b) $\dfrac{1}{(n+1)!} - \dfrac{1}{n!} = \dfrac{1}{(n+1)!} - \dfrac{n+1}{(n+1)!} = \boxed{\dfrac{-n}{(n+1)!}}$

4) $\dfrac{2^n - 2^{n+1}}{2^n - 2^{n-2}} = \dfrac{2^n(1-2)}{2^n(1-2^{-2})} = \dfrac{-1}{1-\frac{1}{4}} = \dfrac{-1}{\frac{3}{4}}$

$= \boxed{-\dfrac{4}{3}}$

## Ex2

1) $f(x) = 3x^2 e^x$

a) $f'(x) = 6x e^x + 3x^2 e^x = \boxed{e^x(6x+3x^2)}$

b) $y = f'(1)(x-1) + f(1)$ $\qquad f(1) = 3e$

$f'(1) = e \times 9 = 9e$

$y = 9e(x-1) + 3e$

$(\Delta) : \boxed{y = 9ex - 6e}$

c) Intersection de $(\Delta)$ avec l'axe $(Ox)$ (donc $y=0$)

On cherche $x$ tel que $9ex - 6e = 0$

$9ex = 6e$

$x = \dfrac{6e}{9e} = \dfrac{6}{9} = \dfrac{2}{3}$

$(\Delta)$ coupe l'axe $(Ox)$ en $\boxed{x = \dfrac{2}{3}}$

2) $g(x) = 4x e^x$

a) $f(x) - g(x) = 3x^2 e^x - 4x e^x = (3x^2 - 4x)e^x$

$\forall x \in \mathbb{R} \quad e^x > 0$

Signe de $3x^2 - 4x$ ? trinôme

$3x^2 - 4x = 0$

$x(3x-4) = 0$

Racines : $x = 0$ $\quad x = \dfrac{4}{3}$

| $x$ | $-\infty$ | | $0$ | | $4/3$ | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $f(x)-g(x)$ | | $+$ | $0$ | $-$ | $0$ | $+$ | |

$a = 3 > 0$

$\mathcal{C}_f$ est en-dessous de $\mathcal{C}_g$ quand $f(x) \leq g(x)$

c'est à dire quand $f(x) - g(x) \leq 0$

donc sur $[0 ; \frac{4}{3}]$

$\boxed{\text{Ex 3}}$ $U_0 = 1$ $U_{n+1} = \dfrac{2U_n - 4}{U_n + 6}$ $V_n = \dfrac{1}{U_n + 2}$

1) $U_1 = \dfrac{2U_0 - 4}{U_0 + 6} = \boxed{\dfrac{-2}{7}}$

$U_2 = \dfrac{2U_1 - 4}{U_1 + 6} = \dfrac{2 \times \frac{-2}{7} - 4}{-\frac{2}{7} + 6} = \dfrac{-\frac{4}{7} - \frac{28}{7}}{-\frac{2}{7} + \frac{42}{7}} = \dfrac{-\frac{32}{7}}{+\frac{40}{7}}$

$U_2 = \dfrac{-32}{40} = \dfrac{-16}{20} = \boxed{\dfrac{-4}{5}}$

2) $V_{n+1} = \dfrac{1}{U_{n+1} + 2} = \dfrac{1}{\frac{2U_n - 4}{U_n + 6} + 2} = \dfrac{1}{\frac{2U_n - 4 + 2(U_n + 6)}{U_n + 6}}$

$= \dfrac{1}{\frac{4U_n + 8}{U_n + 6}} = \dfrac{U_n + 6}{4U_n + 8}$

$V_{n+1} - V_n = \dfrac{U_n + 6}{4U_n + 8} - \dfrac{1}{U_n + 2} = \dfrac{U_n + 6 - 4}{4U_n + 8} = \dfrac{U_n + 2}{4U_n + 8}$

$= \dfrac{U_n + 2}{4(U_n + 2)} = \dfrac{1}{4}$

$V_{n+1} - V_n = \dfrac{1}{4}$ donc $\boxed{V_{n+1} = V_n + \dfrac{1}{4}}$

donc $(V_n)$ est arithmétique de raison $\dfrac{1}{4}$

et de premier terme $V_0 = \dfrac{1}{U_0 + 2} = \boxed{\dfrac{1}{3}}$

3) $V_n = V_0 + n r$

$\boxed{V_n = \dfrac{1}{3} + \dfrac{n}{4}}$

4) $V_n = \dfrac{1}{U_n + 2}$ donc $U_n + 2 = \dfrac{1}{V_n}$

$U_n = \dfrac{1}{V_n} - 2$

avec $V_n = \dfrac{1}{3} + \dfrac{n}{4} = \dfrac{4 + 3n}{12}$

donc $U_n = \dfrac{12}{4 + 3n} - 2 = \dfrac{12 - 8 - 6n}{4 + 3n} = \boxed{\dfrac{4 - 6n}{4 + 3n}}$

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