n°114 p54

$U_0 = \frac{1}{2}$ $\quad \forall n \geq 0 \quad U_{n+1} = \frac{3U_n}{1+2U_n}$ $\quad$ et $0 < U_n < 1$

1) $U_1 = \frac{3U_0}{1+2U_0} = \frac{\frac{3}{2}}{1+1} = \frac{3}{2} \times \frac{1}{2} = \boxed{\frac{3}{4}}$

$U_2 = \frac{3U_1}{1+2U_1} = \frac{\frac{9}{4}}{1+\frac{6}{4}} = \frac{\frac{9}{4}}{\frac{10}{4}} = \boxed{\frac{9}{4}}$

2) $n \geq 0 \quad V_n = \frac{U_n}{1-U_n}$

a) $V_{n+1} = \frac{U_{n+1}}{1-U_{n+1}} = \frac{\frac{3U_n}{1+2U_n}}{1-\frac{3U_n}{1+2U_n}} = \frac{\frac{3U_n}{1+2U_n}}{\frac{1+2U_n-3U_n}{1+2U_n}} = \frac{3U_n}{1-U_n}$

$V_{n+1} = 3 \times \frac{U_n}{1-U_n} = 3V_n$ donc (V_n) est géométrique de raison 3

et de 1er terme $V_0 = \frac{U_0}{1-U_0} = \frac{\frac{1}{2}}{\frac{1}{2}}$ $\boxed{V_0 = 1}$

b) $\forall n \geq 0 \quad V_n = V_0 \times q^n$

$\boxed{V_n = 3^n}$

c) $V_n = \frac{U_n}{1-U_n}$ donc $\frac{U_n}{1-U_n} = 3^n$

$U_n = 3^n(1-U_n)$

$U_n = 3^n - 3^n U_n$

$U_n + 3^n U_n = 3^n$

$U_n(1+3^n) = 3^n$

$\boxed{U_n = \frac{3^n}{3^n+1}}$

d) $U_n = \frac{\cancel{3^n} \times 1}{\cancel{3^n}\left(1+\frac{1}{3^n}\right)}$

$U_n = \frac{1}{1+\left(\frac{1}{3}\right)^n}$ $\qquad \lim_{n \to +\infty} \left(\frac{1}{3}\right)^n = 0$ $\quad$ car $0 < \frac{1}{3} < 1$

donc $\lim_{n \to +\infty} U_n = \frac{1}{1+0} = \boxed{1}$