

Ex 1

$$A = \ln(e^3) = \boxed{3}$$

$$B = \ln(e^{-4}) = \boxed{-4}$$

$$\begin{aligned} C &= \ln\left(\frac{1}{2e^5}\right) = -\ln(2e^5) = -[\ln(2) + \ln(e^5)] \\ &= -[\ln(2) + 5] \\ &= \boxed{-\ln 2 - 5} \end{aligned}$$

$$\begin{aligned} D &= \ln(\sqrt{5}+2) + \ln(\sqrt{5}-2) \\ &= \ln((\sqrt{5}+2)(\sqrt{5}-2)) \\ &= \ln(\sqrt{5}^2 - 2^2) \\ &= \ln(5-4) \\ &= \ln(1) \\ D &= \boxed{0} \end{aligned}$$

Ex 2

$$A = \ln(x) + \ln\left(\frac{1}{x}\right) = \ln(x \times \frac{1}{x}) = \ln(1) = \boxed{0}$$

$$\text{ou } A = \ln(x) - \ln(x) = \boxed{0}$$

$$B = 3\ln(x) - \ln(y) = \ln(x^3) - \ln(y) = \boxed{\ln\left(\frac{x^3}{y}\right)}$$

$$\begin{aligned} C &= -\ln(x) - \ln(y) = -[\ln(x) + \ln(y)] = -[\ln(xy)] \\ \text{ou } C &= \ln\left(\frac{1}{xy}\right) = \ln\left(\frac{1}{y}\right) = \boxed{-\ln(xy)} \\ &= \ln\left(\frac{1}{x} \times \frac{1}{y}\right) \\ &= \boxed{\ln\left(\frac{1}{xy}\right)} = \boxed{-\ln(xy)} \end{aligned}$$

$$\begin{aligned} D &= \ln(x) - \ln(x^2+x) \\ &= \ln\left(\frac{x}{x^2+x}\right) = \boxed{\ln\left(\frac{1}{x+1}\right)} = \boxed{-\ln(x+1)} \\ \text{ou } D &= \ln(x) - \ln(x(x+1)) \\ &= \ln(x) - (\ln(x) + \ln(x+1)) \quad \leftarrow \begin{array}{l} \text{possible car } x > 0 \\ \text{d'après} \\ \ln(x) \end{array} \\ &= \ln(x) - \ln(x) - \ln(x+1) \\ &= \boxed{-\ln(x+1)} \end{aligned}$$

$$E = \ln(x+1) \quad \Delta \text{ Pas de propriété pour } \ln(a+b)$$

$$\begin{aligned} F &= \ln\left(\frac{1}{x}+x\right) - \ln(x) \\ &= \ln\left(\frac{\frac{1}{x}+x}{x}\right) = \ln\left(\frac{1+x^2}{x}\right) = \ln\left(\frac{1+x^2}{x^2}\right) \\ &= \boxed{\ln\left(\frac{1+x^2}{x^2}+1\right)} \end{aligned}$$

$$G = \ln(x) \times \ln(y) \quad \Delta \text{ Pas de propriété pour } \ln(a) \times \ln(b)$$

$$\begin{aligned} H &= \ln(x) \times \ln(x^2) \\ &= \ln(x) \times 2\ln(x) = \boxed{2(\ln(x))^2} \end{aligned}$$

$$I = \frac{\ln(x)}{\ln(y)} \quad \Delta \text{ Pas de propriété pour } \frac{\ln(a)}{\ln(b)}$$

$$J = \frac{\ln(x)}{\ln(x^2)} = \frac{\ln(x)}{2\ln(x)} = \boxed{\frac{1}{2}}$$

Ex 3

$$\begin{aligned} A &= \ln(x^2-1) - 2\ln(x-1) \\ &= \ln(x^2-1) - \ln((x-1)^2) = \ln\left(\frac{x^2-1}{(x-1)^2}\right) \\ &= \ln\left(\frac{(x-1)(x+1)}{(x-1)^2}\right) = \boxed{\ln\left(\frac{x+1}{x-1}\right)} \end{aligned}$$

$$\begin{aligned} B &= \frac{1}{2}\ln(x+2) + 3\ln(x) \\ &= \ln(\sqrt{x+2}) + \ln(x^3) = \boxed{\ln(x^3\sqrt{x+2})} \end{aligned}$$

$$\begin{aligned} C &= -\ln\left(\frac{x}{3}\right) - 5\ln(x) \\ &= \ln\left(\frac{3}{x}\right) - \ln(x^5) = \ln\left(\frac{\frac{3}{x}}{x^5}\right) = \boxed{\ln\left(\frac{3}{x^6}\right)} \end{aligned}$$

$$\begin{aligned} \text{ou } C &= -\ln\left(\frac{x}{3}\right) - 5\ln(x) \\ &= -\ln\left(\frac{x}{3}\right) - \ln(x^5) \\ &= -[\ln\left(\frac{x}{3}\right) + \ln(x^5)] \\ &= -[\ln\left(\frac{x}{3} \times x^5\right)] = \boxed{-\ln\left(\frac{x^6}{3}\right)} \\ &= \boxed{\ln\left(\frac{3}{x^6}\right)} \end{aligned}$$

Ex 4 $\forall x \in \mathbb{R} \quad f(x) = \ln((x+1)^2) - \ln((x-1)^2)$

$$\begin{aligned} f(-x) &= \ln((-x+1)^2) - \ln((-x-1)^2) \\ &= \ln((1-x)^2) - \ln((-1)(x+1)^2) \\ &= \ln((1-x)^2) - \ln((x+1)^2) \end{aligned}$$

$f(-x) = -f(x)$

Résp: f est impaire
et dans un repère orthogonal du plan, C_f est
symétrique par rapport à l'origine.

Ex 5 $U_0 = 1 \quad \forall n \geq 1 \quad U_{n+1} = e^2 U_n$

 $V_n = \ln(U_n)$

1) $V_{n+1} = \ln(U_{n+1}) = \ln(e^2 U_n) = \ln(e^2) + \ln(U_n)$
 $= 2 + \ln(U_n)$

donc $V_{n+1} - V_n = 2 + \ln(U_n) - \ln(U_n) = 2$

donc $V_{n+1} = V_n + 2$ pour tout $n \geq 0$

et donc (V_n) est arithmétique de raison 2
et de premier terme $V_0 = \ln(U_0) = \ln(1) = 0$.

2) Pour tout $n \geq 0 \quad V_n = V_0 + n \cdot 2$

$V_n = 2n$

3) $V_n = \ln(U_n)$ D'après $\ln(x) = a \iff x = e^a$
donc $U_n = e^{V_n}$
 $U_n = e^{2n}$

Vérification:
Avec $U_0 = 1$ et $U_{n+1} = e^2 U_n$ on a
 $U_0 = 1 \quad \leftarrow e^{2 \times 0}$
 $U_1 = e^2 U_0 = e^2 \quad \leftarrow e^{2 \times 1}$
 $U_2 = e^2 U_1 = e^2 \times e^2 = e^4 \quad \leftarrow e^{2 \times 2}$
 $U_3 = e^2 U_2 = e^2 \times e^4 = e^6 \quad \leftarrow e^{2 \times 3}$

Ex 6 $U_0 = 1 \quad \forall n \geq 0 \quad U_{n+1} = \frac{U_n^2}{5}$

 $V_n = \ln\left(\frac{U_n}{5}\right)$

1) $V_{n+1} = \ln\left(\frac{U_{n+1}}{5}\right) = \ln\left(\frac{U_n^2}{5^2}\right) = \ln\left(\left(\frac{U_n}{5}\right)^2\right) = 2 \ln\left(\frac{U_n}{5}\right) = 2 V_n$

donc $V_{n+1} = 2 V_n$ pour tout $n \geq 0$

donc (V_n) est géométrique de raison 2 de premier terme $V_0 = \ln\left(\frac{U_0}{5}\right) = \ln\left(\frac{1}{5}\right) = -\ln(5)$

2) $\forall n \geq 0 \quad V_n = V_0 \times q^n$
 $V_n = -\ln(5) \times 2^n$ ou $V_n = \ln\left(\frac{1}{5}\right) \times 2^n$

3) $V_n = \ln\left(\frac{U_n}{5}\right)$ D'après $\ln(x) = a \iff x = e^a$
donc $\frac{U_n}{5} = e^{V_n}$
 $U_n = 5 e^{\ln\left(\frac{1}{5}\right) \times 2^n}$
 $U_n = 5 e^{\ln\left(\frac{1}{5}\right) 2^n}$
 $U_n = 5 \left[e^{\ln\left(\frac{1}{5}\right)}\right]^{2^n}$
 $U_n = 5 \left[\frac{1}{5}\right]^{2^n}$
 $U_n = \frac{5}{5^{2^n}} = \frac{1}{5^{2^n-1}}$

Vérification:

$U_0 = 1$	$n = 0 \quad \frac{1}{5^{2^0-1}} = \frac{1}{5^{1-1}} = \frac{1}{5^0} = 1$
$U_1 = \frac{U_0^2}{5} = \frac{1}{5}$	$n = 1 \quad \frac{1}{5^{2^1-1}} = \frac{1}{5^{2-1}} = \frac{1}{5^1} = \frac{1}{5}$
$U_2 = \frac{U_1^2}{5} = \frac{1}{5^3}$	$n = 2 \quad \frac{1}{5^{2^2-1}} = \frac{1}{5^{4-1}} = \frac{1}{5^3} = \frac{1}{125}$
$U_3 = \frac{U_2^2}{5} = \frac{1}{5^7}$	$n = 3 \quad \frac{1}{5^{2^3-1}} = \frac{1}{5^{8-1}} = \frac{1}{5^7} = \frac{1}{78125}$