DM Calculs 2

Ex 1 $\forall n \geq 0 \quad \begin{cases} U_{n+1} = 3 - \dfrac{10}{U_n + 4} \\ U_0 = 5 \end{cases}$

$\forall n \geq 0 \quad V_n = \dfrac{U_n - 1}{U_n + 2}$

Astuce !

1) a) $V_{n+1} = \dfrac{U_{n+1} - 1}{U_{n+1} + 2}$

$= \dfrac{3 - \dfrac{10}{U_n+4} - 1}{3 - \dfrac{10}{U_n+4} + 2} = \dfrac{\dfrac{2(U_n+4) - 10}{U_n+4}}{\dfrac{5(U_n+4)-10}{U_n+4}}$ $\times (U_n+4)$ $\times (U_n+4)$

$= \dfrac{2(U_n+4)-10}{5(U_n+4)-10} = \dfrac{2U_n - 2}{5U_n + 10} = \dfrac{2(U_n - 1)}{5(U_m + 2)}$

$= \dfrac{2}{5} \times \dfrac{U_n - 1}{U_n + 2}$

$$\boxed{V_{n+1} = \frac{2}{5} V_n}$$

donc (V_n) est géométrique de raison $\frac{2}{5}$ de premier terme

$V_0 = \dfrac{U_0 - 1}{U_0 + 2} = \dfrac{4}{7}$

b) $\forall n \in \mathbb{N} \quad V_n = V_0 \times q^n$

$$\boxed{V_n = \frac{4}{7} \times \left(\frac{2}{5}\right)^n}$$

2) $V_n = \dfrac{U_n - 1}{U_n + 2}$

$\iff (U_n + 2) V_n = U_n - 1$

$U_n V_n + 2V_n = U_n - 1$

$U_n V_n - U_n = -2V_n - 1$

$U_n(V_n - 1) = -2V_n - 1$

$V_n \neq 1$ donc $\boxed{U_n = \dfrac{-2V_n - 1}{V_n - 1}}$

$\frac{4}{7} < 1$

$\frac{2}{5} < 1$ donc $\forall n \in \mathbb{N}$ $\left(\frac{2}{5}\right)^n < 1$

Conclusion : $\forall n \in \mathbb{N}$ $\frac{4}{7} \times \left(\frac{2}{5}\right)^n < 1$

$\forall n \in \mathbb{N}$ $V_n < 1$

donc $V_n \neq 1$