

Tous les résultats doivent être donnés simplifiés

$$\alpha \quad 1. \text{ Simplifier : } \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = \boxed{2\sqrt{5}}$$

$$\alpha \quad 2. \text{ Ecrire sous la forme } \sqrt{a} : 5\sqrt{3} = \sqrt{25} \times \sqrt{3} = \boxed{\sqrt{75}}$$

3. Calculer et donner le résultat sous la forme la plus simple.

$$\alpha \quad 2\sqrt{3} \times 3\sqrt{7} = \boxed{6\sqrt{21}}$$

$$\alpha \quad 2\sqrt{3} + \sqrt{3} = \boxed{3\sqrt{3}}$$

$$\alpha \quad (4\sqrt{5})^2 = 4^2 \times (\sqrt{5})^2 = 16 \times 5 = \boxed{80}$$

$$(3\sqrt{2})^3 = 3^3 \times \sqrt{2}^3 = 27 \times (\sqrt{2})^2 \times \sqrt{2} = \boxed{54\sqrt{2}}$$

$$\frac{\sqrt{5^8}}{25} = \frac{5^4}{5^2} = 5^2 = \boxed{25}$$

$$\sqrt{3^7} = \sqrt{3^6 \times 3} = \sqrt{3^6} \times \sqrt{3} = 3^3 \times \sqrt{3} = \boxed{27\sqrt{3}}$$

$$3\sqrt{2}(\sqrt{2} - 4\sqrt{3}) = 3(\sqrt{2})^2 - 12\sqrt{6} = \boxed{6 - 12\sqrt{6}}$$

$$\frac{\sqrt{\frac{5}{3}}}{\sqrt{3}} = \frac{\frac{\sqrt{5}}{\sqrt{3}}}{\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{5}}{3}}$$

$$\begin{aligned} \sqrt{50} + \sqrt{8} &= \sqrt{25 \times 2} + \sqrt{4 \times 2} \\ &= \sqrt{25} \times \sqrt{2} + \sqrt{4} \times \sqrt{2} \\ &= 5\sqrt{2} + 2\sqrt{2} = \boxed{7\sqrt{2}} \end{aligned}$$

$$\sqrt{3\sqrt{25}+\sqrt{9}} = \sqrt{3 \times 5 + 3} = \sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = \boxed{3\sqrt{2}}$$

$$\begin{aligned} (\sqrt{5}-\sqrt{7})^2 &= (\sqrt{5})^2 - 2 \times \sqrt{5} \times \sqrt{7} + (\sqrt{7})^2 \\ &= 5 - 2\sqrt{35} + 7 \\ &= \boxed{12 - 2\sqrt{35}} \end{aligned}$$

$$\begin{aligned} 5 - (3\sqrt{2} + \sqrt{3})^2 &= 5 - ((3\sqrt{2})^2 + 2 \times 3\sqrt{2} \times \sqrt{3} + (\sqrt{3})^2) \\ &= 5 - (18 + 6\sqrt{6} + 3) \\ &= 5 - (21 + 6\sqrt{6}) = 5 - 21 - 6\sqrt{6} = \boxed{-16 - 6\sqrt{6}} \end{aligned}$$

4. Supprimer la racine carrée au dénominateur :

$$\frac{2}{\sqrt{7}} = \frac{2 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \boxed{\frac{2\sqrt{7}}{7}}$$

$$\begin{aligned} \frac{2}{4-\sqrt{2}} &= \frac{2(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})} = \frac{8+2\sqrt{2}}{16-2} = \frac{8+2\sqrt{2}}{14} = \frac{2(4+\sqrt{2})}{2 \times 7} \\ &= \boxed{\frac{4+\sqrt{2}}{7}} \end{aligned}$$

5. Soit $f(x) = (x-1)(x^2+2)$ pour $x \in \mathbb{R}$.

a. Calculer $f(-\sqrt{2})$.

$$\begin{aligned} f(-\sqrt{2}) &= (-\sqrt{2}-1)((-\sqrt{2})^2+2) \\ &= (-\sqrt{2}-1)(4) \\ &= \boxed{-4\sqrt{2}-4} \end{aligned}$$

b. Calculer $f(1+\sqrt{3})$.

$$\begin{aligned} f(1+\sqrt{3}) &= (1+\sqrt{3}-1)((1+\sqrt{3})^2+2) \\ &= \sqrt{3}(1+2\sqrt{3}+3+2) \\ &= \sqrt{3}(6+2\sqrt{3}) \\ &= 6\sqrt{3} + 2(\sqrt{3})^2 = \boxed{6\sqrt{3} + 6} \end{aligned}$$