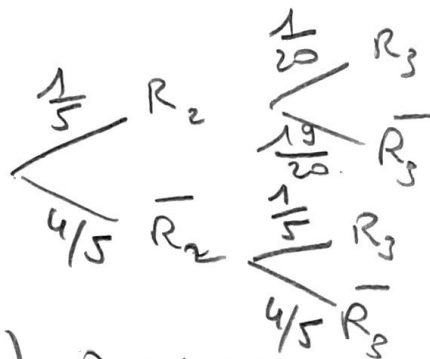


Ex 1

A

$$1) P(\bar{R}_1 \cap \bar{R}_2 \cap \bar{R}_3) = P(\bar{R}_2 \cap \bar{R}_3)$$

$$= \frac{4}{5} \times \frac{4}{5} = \boxed{\frac{16}{25}}$$



$$2) P(R_3) = P(R_2 \cap R_3) + P(\bar{R}_2 \cap R_3) \quad \text{Prob. totales}$$

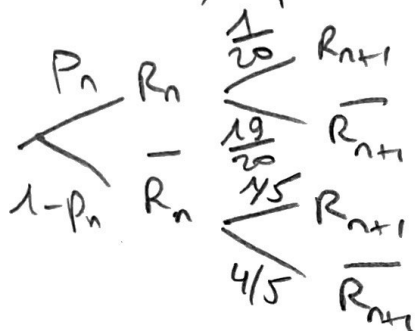
$$= P(R_2) \times P(R_3) + P(\bar{R}_2) \times P(R_3)$$

$$= \frac{1}{5} \times \frac{1}{20} + \frac{4}{5} \times \frac{1}{5} = \frac{1}{100} + \frac{4}{25} = \boxed{\frac{9}{100}}$$

$$3) P_{R_3}(R_2) = \frac{P(R_2 \cap R_3)}{P(R_3)} = \frac{\frac{1}{5} \times \frac{1}{20}}{\frac{9}{100}} = \frac{\frac{1}{100}}{\frac{9}{100}} = \boxed{\frac{1}{9}}$$

B

$$P_n = P(R_n) \quad P_1 = 0$$



$$\begin{aligned} P_{n+1} &= P(R_{n+1}) \\ &= P(R_n \cap R_{n+1}) + P(\bar{R}_n \cap R_{n+1}) \\ &= P_n \times \frac{1}{20} + (1 - P_n) \times \frac{1}{5} \\ &= \frac{P_n}{20} + \frac{1}{5} - \frac{P_n}{5} \\ &= -\frac{3P_n}{20} + \frac{1}{5} \end{aligned}$$

$$\boxed{P_{n+1} = \frac{1}{5} - \frac{3}{20} P_n}$$

$$2) V_n = P_n - \frac{4}{23}$$

$$V_{n+1} = P_{n+1} - \frac{4}{23} = \frac{1}{5} - \frac{4}{23} - \frac{3}{20} P_n = \frac{3}{115} - \frac{3}{20} P_n$$

$$V_{n+1} = -\frac{3}{20} \left(\frac{3}{115} + P_n \right)$$

$$V_{n+1} = -\frac{3}{20} \left(-\frac{20}{115} + P_n \right) = -\frac{3}{20} \left(-\frac{4}{23} + P_n \right)$$

$$\boxed{V_{n+1} = -\frac{3}{20} V_n}$$

(Vn) geom de raison $-\frac{3}{20}$

de 1er terme $V_1 = P_1 - \frac{4}{23} = -\frac{4}{23}$

$$V_n = V_1 \times q^n = -\frac{4}{23} \times \left(-\frac{3}{20}\right)^n$$

$$P_n = V_n + \frac{4}{23}$$

$$P_n = -\frac{4}{23} \left(-\frac{3}{20}\right)^n + \frac{4}{23} = \frac{4}{23} \left(1 - \left(-\frac{3}{20}\right)^n\right)$$

$$\boxed{\lim P_n = \frac{4}{23}}$$