

Limite de fonctions composées - Correction

1. Calcul de $\lim_{x \rightarrow -\infty} \sqrt{x^2 + 7}$

On a $\lim_{x \rightarrow -\infty} x^2 + 7 = +\infty$ donc $\lim_{x \rightarrow -\infty} \sqrt{x^2 + 7} = \lim_{X \rightarrow +\infty} \sqrt{X} = +\infty$

2. Calcul de $\lim_{x \rightarrow +\infty} (-2\sqrt{x} + 3)^2$

On a $\lim_{x \rightarrow +\infty} -2\sqrt{x} + 3 = -\infty$ donc $\lim_{x \rightarrow +\infty} (-2\sqrt{x} + 3)^2 = \lim_{X \rightarrow -\infty} X^2 = +\infty$

3. Calcul de $\lim_{x \rightarrow +\infty} \frac{e^{1-3x}}{2x}$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} e^{1-3x} = \lim_{X \rightarrow -\infty} e^X = 0 \\ \lim_{x \rightarrow +\infty} 2x = +\infty \end{array} \right\} \begin{array}{l} \text{Par quotient :} \\ \lim_{x \rightarrow +\infty} \frac{e^{1-3x}}{2x} = 0 \end{array}$$

4. Calcul de $\lim_{x \rightarrow -\infty} (2x-1)\sqrt{1-x}$

$$\left. \begin{array}{l} \lim_{x \rightarrow -\infty} 2x-1 = -\infty \\ \lim_{x \rightarrow -\infty} \sqrt{1-x} = \lim_{X \rightarrow +\infty} \sqrt{X} = +\infty \end{array} \right\} \begin{array}{l} \text{Par produit :} \\ \lim_{x \rightarrow -\infty} (2x-1)\sqrt{1-x} = -\infty \end{array}$$

5. Calcul de $\lim_{x \rightarrow -\infty} (2 - e^x)^3$

On a $\lim_{x \rightarrow -\infty} e^x = 0$ donc $\lim_{x \rightarrow -\infty} 2 - e^x = 2$

donc $\lim_{x \rightarrow -\infty} (2 - e^x)^3 = \lim_{X \rightarrow 2} X^3 = 2^3 = 8$ car la fonction $x \mapsto x^3$ est continue sur $\mathbb{R}$ donc en 2.

6. Pour $x < 2$, $\lim_{x \rightarrow 2} \exp\left(\frac{1-x}{4-2x}\right)$

$$\left. \begin{array}{l} \lim_{x \rightarrow 2} 1-x = -1 \\ \lim_{x \rightarrow 2} 4-2x = 0^+ \end{array} \right\} \begin{array}{l} \text{Par quotient :} \\ \lim_{x \rightarrow 2} \frac{1-x}{4-2x} = -\infty \end{array}$$

Justification du signe de $4-2x$:

On a $x < 2$ donc $-2x > -4$ et donc $4-2x > 0$

Conclusion : $\lim_{x \rightarrow 2} \exp\left(\frac{1-x}{4-2x}\right) = \lim_{X \rightarrow -\infty} e^X = 0$