Sujet B

Ex 1

① $f(x) = \dfrac{2}{(3e^x+5)^3} = 2(3e^x+5)^{-3}$

$(u^{-3})' = -3u^{-4} \times u'$

$f'(x) = 2 \times (-3)(3e^x+5)^{-4} \times 3e^x$

$f'(x) = -6 \times \dfrac{1}{(3e^x+5)^4} \times 3e^x = \boxed{\dfrac{-18e^x}{(3e^x+5)^4}}$

② $f(x) = \cos(x)\sin^3(x)$

$f = uv$ et $v = w^3$

$f' = u'v + uv'$ | $v' = 3w \times w'$

$f'(x) = -\sin(x) \times \sin^3(x) + \cos(x) \times 3\sin^2(x) \times \cos(x)$

$\boxed{f'(x) = -\sin^4(x) + 3\cos^2(x)\sin^2(x)}$

ou $f'(x) = \sin^2(x)\left(-\sin^2(x) + 3\cos^2(x)\right)$

③ $f(x) = \dfrac{e^{3x}}{x^2}$

$f = \dfrac{u}{v}$ et $u = e^w$

$f' = \dfrac{u'v - uv'}{v^2}$ $u' = e^w \times w'$

$f'(x) = \dfrac{3e^{3x} \times x^2 - e^{3x} \times 2x}{(x^2)^2}$

$f'(x) = \dfrac{e^{3x}(3x^2-2x)}{x^4} = \dfrac{e^{3x} x (3x-2)}{x^4} = \boxed{\dfrac{e^{3x}(3x-2)}{x^3}}$

Ex 2

1) $f(x) = -2x^3 + \dfrac{5}{\sqrt{x}} - 2$

$F(x) = -2 \times \dfrac{x^4}{4} + 5 \times 2\sqrt{x} - 2x$

$\boxed{F(x) = -\frac{1}{2}x^4 + 10\sqrt{x} - 2x}$

2) $f(x) = 3e^{4x-1}$

$f(x) = \dfrac{3}{4} \times 4e^{4x-1}$

$\boxed{F(x) = \frac{3}{4}e^{4x-1}}$

$\dfrac{u'e^u}{}$ Primitive e^u

$f = k \times u'e^u$

avec $u(x) = 4x-1$

$u'(x) = 4$

3) $f(x) = \dfrac{4x^2}{\sqrt{x^3+1}}$

$f(x) = \dfrac{4}{3} \times \dfrac{3x^2}{\sqrt{x^3+1}}$

$F(x) = \dfrac{4}{3} \times 2\sqrt{x^3+1} = \boxed{\dfrac{8}{3}\sqrt{x^3+1}}$

$\dfrac{u'}{\sqrt{u}}$ Primitive $2\sqrt{u}$

$f = k \times \dfrac{u'}{\sqrt{u}}$

avec $u(x) = x^3+1$

$u'(x) = 3x^2$

4) $f(x) = \dfrac{1}{2x^2}\left(1+\dfrac{1}{x}\right)^3$

$f(x) = -\dfrac{1}{2} \times \dfrac{-1}{x^2}\left(1+\dfrac{1}{x}\right)^3$

$F(x) = -\dfrac{1}{2} \times \dfrac{\left(1+\frac{1}{x}\right)^4}{4}$

$\boxed{F(x) = -\frac{1}{8}\left(1+\frac{1}{x}\right)^4}$

$u'u^3$ Primitive $\dfrac{u^4}{4}$

$f = k \times u'u^3$

avec $u(x) = 1+\dfrac{1}{x}$

$u'(x) = -\dfrac{1}{x^2}$

Ex 3 $f(x) = \dfrac{5x}{(3x^2+1)^2}$

$f(x) = \dfrac{5}{6} \times \dfrac{6x}{(3x^2+1)^2}$

$F(x) = \dfrac{5}{6} \times \dfrac{-1}{3x^2+1} + C$ avec $C \in \mathbb{R}$

$F(0) = \dfrac{5}{6} \times \dfrac{-1}{1} + C = -\dfrac{5}{6} + C$

$F(0) = 2 \iff -\dfrac{5}{6} + C = 2$

$\iff C = 2 + \dfrac{5}{6}$

$\iff C = \dfrac{17}{6}$

donc $\boxed{F(x) = -\dfrac{5}{6(3x^2+1)} + \dfrac{17}{6}.}$

$\dfrac{u'}{u^2}$ Primitive $-\dfrac{1}{u}$

$f = k \times \dfrac{u'}{u^2}$

avec $u(x) = 3x^2+1$

$u'(x) = 6x$