Ex 1

① $f(x) = \cos(x)\sin^2(x)$ | $f = uv$ avec $v = w^2$, $f' = u'v + uv'$, $v' = 2w \times w'$

$f'(x) = -\sin(x)\sin^2(x) + \cos(x) \times 2\sin(x) \times \cos(x)$

$f'(x) = -\sin^3(x) + 2\sin(x)\cos^2(x)$

ou $f'(x) = \sin(x)\left(-\sin^2(x) + 2\cos^2 x\right)$

② $f(x) = \dfrac{e^{2x}}{x^3}$ | $f = \dfrac{u}{v}$ avec $u = e^w$, $f' = \dfrac{u'v - uv'}{v^2}$, $u' = e^w \times w'$

$f'(x) = \dfrac{e^{2x} \times 2 \times x^3 - e^{2x} \times 3x^2}{(x^3)^2} = \dfrac{e^{2x}(2x^3 - 3x^2)}{x^6}$

$f'(x) = \dfrac{e^{2x} \times x^2(2x-3)}{x^6} = \boxed{\dfrac{e^{2x}(2x-3)}{x^6}}$

③ $f(x) = \dfrac{4}{(e^x+2)^3} = 4(e^x+2)^{-3}$ $\quad (u^{-3})' = -3u^{-4} \times u'$

$f'(x) = 4 \times (-3)(e^x+2)^{-4} \times e^x$

$f'(x) = \dfrac{-12e^x}{(e^x+2)^4}$

5

12 11 25

Ex 2 1) $f(x) = 5x^3 - \dfrac{3}{\sqrt{x}} + 4$

$F(x) = 5 \times \dfrac{x^4}{4} - 3 \times 2\sqrt{x} + 4x$

$F(x) = \dfrac{5}{4}x^4 - 6\sqrt{x} + 4x$

2) $f(x) = 2e^{1-x}$

$f(x) = -2 \times (-1)e^{1-x}$

$F(x) = -2e^{1-x}$

Utiliser : $u'e^u$
Primitive e^u
Ecrit $f = k \times u'e^u$
avec $u(x) = 1-x$
$u'(x) = -1$

3) $f(x) = \dfrac{3x}{(x^2-1)^2}$

$f(x) = \dfrac{3}{2} \times \dfrac{2x}{(x^2-1)^2}$

$F(x) = \dfrac{3}{2} \times \dfrac{-1}{x^2-1}$

$F(x) = \dfrac{-3}{2(x^2-1)}$

Utiliser $\dfrac{u'}{u^2}$
Primitive $-\dfrac{1}{u}$
Ecrit $f = k \times \dfrac{u'}{u^2}$
avec $u(x) = x^2-1$
$u'(x) = 2x$

4) $f(x) = \dfrac{1}{5x^2}\left(2 + \dfrac{1}{x}\right)^3$

$f(x) = -\dfrac{1}{5} \times \dfrac{-1}{x^2}\left(2 + \dfrac{1}{x}\right)^3$

$F(x) = -\dfrac{1}{5} \times \dfrac{\left(2 + \frac{1}{x}\right)^4}{4}$

$F(x) = -\dfrac{1}{20}\left(2 + \dfrac{1}{x}\right)^4$

Utiliser $u'u^3$
Primitive $\dfrac{u^4}{4}$ ou $\dfrac{1}{4}u^4$
Ecrit $f = k \times u'u^3$
avec $u(x) = 2 + \dfrac{1}{x}$
$u'(x) = -\dfrac{1}{x^2}$

Ex 3 Sur $\mathbb{R}$, $f(x) = \dfrac{x}{\sqrt{3x^2+1}}$

$f(x) = \dfrac{1}{6} \times \dfrac{6x}{\sqrt{3x^2+1}}$

$F(x) = \dfrac{1}{6} \times 2\sqrt{3x^2+1} + C = \dfrac{1}{3}\sqrt{3x^2+1} + C$

$F(-1) = \dfrac{1}{3}\sqrt{4} + C = \dfrac{2}{3} + C$ avec $C \in \mathbb{R}$

$F(-1) = 3 \iff \dfrac{2}{3} + C = 3 \iff C = 3 - \dfrac{2}{3} \iff C = \dfrac{7}{3}$

donc $F(x) = \dfrac{1}{3}\sqrt{3x^2+1} + \dfrac{7}{3}$

Utiliser $\dfrac{u'}{\sqrt{u}}$
Primitive $2\sqrt{u}$
Ecrit $f = k \times \dfrac{u'}{\sqrt{u}}$
avec $u(x) = 3x^2+1$
$u'(x) = 6x$

12 11 25