Ex1 Calculer U_{n+1} dans les cas suivants

1) $U_n = \frac{3}{2}n^2 - n$ pour $n \geq 0$

$U_{n+1} = \frac{3}{2}(n+1)^2 - (n+1) = \frac{3}{2}(n^2+2n+1) - n - 1$

$= \frac{3}{2}n^2 + 3n + \frac{3}{2} - n - 1 = \boxed{\frac{3}{2}n^2 + 2n + \frac{1}{2}}$

2) $U_n = \frac{2n}{1-n}$ pour $n \geq 2$

$U_{n+1} = \frac{2(n+1)}{1-(n+1)} = \frac{2(n+1)}{1-n-1} = \frac{2(n+1)}{-n} \quad \boxed{\frac{-2(n+1)}{n}}$

3) $U_n = n e^{-n}$ pour $n \geq 0$

$U_{n+1} = (n+1)e^{-(n+1)} = \boxed{(n+1)e^{-n-1}}$

Ex2 Factoriser les expressions suivantes

1) $e^x + (x-2)(-e^x) = e^x(1-(x-2))$

$= e^x(1-x+2)$

$= \boxed{e^x(3-x)}$

2) $e^{2x} - \left(-x+\frac{1}{2}\right)e^x + e^x \times (-2) = e^x\left(e^x - \left(-x+\frac{1}{2}\right) - 2\right)$

$= e^x\left(e^x + x - \frac{1}{2} - 2\right)$

$= \boxed{e^x\left(e^x + x - \frac{5}{2}\right)}$

3) $x^n e^{-x} + x^n = \boxed{x^n(e^{-x}+1)}$

4) $x^{n+1}e^x - x^n e^{2x} = \boxed{x^n e^x(x - e^x)}$

5) $x^{n-1} - 3x^n = \boxed{x^{n-1}(1-3x)}$

6) $\frac{2}{e^{-x}} - 4xe^x = 2e^x - 4xe^x = \boxed{e^x(2-4x)}$

Ex3 Calculer $U_{n+1} - U_n$ dans les cas suivants

1) $U_{n+1} = \frac{1}{2}\left(U_n + \frac{9}{U_n}\right)$

$U_{n+1} - U_n = \frac{1}{2}\left(U_n + \frac{9}{U_n}\right) - U_n = \frac{1}{2}U_n + \frac{9}{2U_n} - U_n$

$= \frac{9}{2U_n} - \frac{1}{2}U_n$

$= \boxed{\frac{9 - U_n^2}{2U_n}} = \boxed{\frac{(3-U_n)(3+U_n)}{2U_n}}$

2) $U_{n+1} = \frac{9}{6-U_n}$.

$U_{n+1} - U_n = \frac{9}{6-U_n} - U_n = \frac{9 - U_n(6-U_n)}{6-U_n}$

$= \boxed{\frac{9 - 6U_n + U_n^2}{6-U_n}} = \boxed{\frac{(3-U_n)^2}{6-U_n}}$

3) $U_{n+1} = \frac{n+1}{2n}U_n$

$U_{n+1} - U_n = \frac{n+1}{2n}U_n - U_n = \frac{(n+1)U_n - 2nU_n}{2n}$

$= U_n\left(\frac{n+1}{2n} - 1\right) \Big\| = \frac{nU_n + U_n - 2nU_n}{2n}$

$= U_n\left(\frac{n+1-2n}{2n}\right) \Big\| = \frac{U_n - nU_n}{2n} = \boxed{\frac{U_n(1-n)}{2n}}$

$= U_n\left(\frac{-n+1}{2n}\right) \Big\|$

4) $U_{n+1} = \frac{nU_n}{2(n+1)}$

$U_{n+1} - U_n = \frac{nU_n}{2(n+1)} - U_n = \frac{nU_n - U_n \times 2(n+1)}{2(n+1)}$

$= U_n\left(\frac{n}{2(n+1)} - 1\right) \Big\| = \frac{U_n(n - 2(n+1))}{2(n+1)}$

$= U_n\left(\frac{n-2(n+1)}{2(n+1)}\right) \Big\| = \frac{U_n(n-2n-2)}{2(n+1)}$

$= U_n\left(\frac{n-2n-2}{2(n+1)}\right) \Big\| = \frac{U_n(-n-2)}{2(n+1)}$

$= U_n\left(\frac{-n-2}{2(n+1)}\right) \Big\|$