

Limites Fonction ln F.I - Correction

1. $\lim_{x \rightarrow +\infty} x^2 - \ln x$ F.I. de la forme $+\infty - \infty$

On a : $x^2 - \ln x = x \left(x - \frac{\ln x}{x} \right)$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} x = \boxed{+\infty} \\ \lim_{x \rightarrow +\infty} \frac{\ln x}{x} = 0 \quad \text{donc} \quad \lim_{x \rightarrow +\infty} x - \frac{\ln x}{x} = \boxed{+\infty} \end{array} \right\} \begin{array}{l} \text{Par produit : } \lim_{x \rightarrow +\infty} x \left(x - \frac{\ln x}{x} \right) = \boxed{+\infty} \\ \text{donc } \boxed{\lim_{x \rightarrow +\infty} x^2 - \ln x = +\infty} \end{array}$$

2. $\lim_{x \rightarrow +\infty} x - 5 \ln x$ F.I. de la forme $+\infty - \infty$

On a : $x - 5 \ln x = x \left(1 - 5 \times \frac{\ln x}{x} \right)$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} x = \boxed{+\infty} \\ \lim_{x \rightarrow +\infty} \frac{\ln x}{x} = 0 \quad \text{donc} \quad \lim_{x \rightarrow +\infty} 1 - 5 \times \frac{\ln x}{x} = \boxed{1} \end{array} \right\} \begin{array}{l} \text{Par produit : } \lim_{x \rightarrow +\infty} x \left(1 - 5 \times \frac{\ln x}{x} \right) = \boxed{+\infty} \\ \text{donc } \boxed{\lim_{x \rightarrow +\infty} x - 5 \ln x = +\infty} \end{array}$$

3. Soit $a < 0$, $\lim_{x \rightarrow +\infty} \ln(x) - ax$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} \ln x = \boxed{+\infty} \\ \lim_{x \rightarrow +\infty} -ax = \boxed{+\infty} \end{array} \right\} \text{Par somme : } \boxed{\lim_{x \rightarrow +\infty} \ln(x) - ax = +\infty}$$

4. Soit $a > 0$, $\lim_{x \rightarrow +\infty} \ln(x) - ax$ F.I. de la forme $+\infty - \infty$

On a : $\ln(x) - ax = x \left(\frac{\ln x}{x} - a \right)$.

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} x = \boxed{+\infty} \\ \lim_{x \rightarrow +\infty} \frac{\ln x}{x} = 0 \quad \text{donc} \quad \lim_{x \rightarrow +\infty} \frac{\ln x}{x} - a = \boxed{-a} \end{array} \right\} \begin{array}{l} \text{Par produit : } \lim_{x \rightarrow +\infty} x \left(\frac{\ln x}{x} - a \right) = \boxed{-\infty} \\ \text{donc } \boxed{\lim_{x \rightarrow +\infty} \ln(x) - ax = -\infty} \end{array}$$

5. $\lim_{x \rightarrow +\infty} \frac{\ln x}{x+2}$ F.I. de la forme $\frac{+\infty}{+\infty}$

On a $\frac{\ln x}{x+2} = \frac{\ln x}{x \left(1 + \frac{2}{x} \right)} = \frac{\ln x}{x} \times \frac{1}{1 + \frac{2}{x}}$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} \frac{\ln x}{x} = \boxed{0} \\ \lim_{x \rightarrow +\infty} 1 + \frac{2}{x} = 1 \quad \text{donc} \quad \lim_{x \rightarrow +\infty} \frac{1}{1 + \frac{2}{x}} = \boxed{1} \end{array} \right\} \begin{array}{l} \text{Par produit : } \lim_{x \rightarrow +\infty} \frac{\ln x}{x} \times \frac{1}{1 + \frac{2}{x}} = \boxed{0} \\ \text{donc } \boxed{\lim_{x \rightarrow +\infty} \frac{\ln x}{x+2} = 0} \end{array}$$